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Simulation of the Pressure Field Near a Jet by Randomly Distributed Vortex Rings

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Fluctuations of the pressure field in the vicinity of a jet are simulated numerically by a flow model consisting of axially symmetric vortex rings with viscous cores submerged in a uniform stream. The time interval between the shedding of successive vortices is taken to be a random variable with a probability distribution chosen to match that from experiments. It is found that up to 5 diameters downstream of the jet exit, statistics of the computed pressure field are in good agreement with experimental results. Statistical comparisons are provided for the overall sound pressure level, the peak amplitude, and the Strouhal number based on the peak frequency of the pressure signals.

I. Introduction

THE concept of modeling turbulent jets in terms of some well-defined patterns has been proposed by many researchers as a way to attack the problem of jet noise generation and its suppression. Based on the experimental observation that pressure fluctuations outside the mixing region come in rather well-organized packages, Mollo-Christensen[1] first suggested that certain natural organized structures might exist within the shear flow turbulence and that these structures might be the primary mechanism for the generation of jet noise. A number of subsequent investigations have focused on this approach with the hope that this regular pattern, if any, and its role in jet noise production could be better defined. Later, Becker and Massaro[2] carried out a study on the vortex evolution and varicose instability of an axially symmetric jet with a Reynolds number in the range of 10^4. Crow and Champagne[3] performed measurements at a Reynolds number around 10^5 and reported that a large-scale, orderly structure was observed within the noise-producing region of a jet. Based on these observations, they proposed a theoretical flow model consisting of an axially symmetric vortex train with a preferred Strouhal number of 0.3 emerging by calculation. This model of an axial array of vortices also has been recommended by other researchers[4-7] as the basic structure of a flow pattern designed to initiate the repetitive and deterministic behavior of turbulent round jets.

There have been numerous studies of vortex rings; however, most of these concentrate only on a single vortex. The classical vortex ring of small cross section in a perfect fluid[8] was physically unsatisfactory until the viscous effects were taken into account. By considering an inner viscous core and applying the matched asymptotic expansions technique, Tung and Ting[9] removed the mathematical singularity, which is the central drawback of the invisicid model. The same solution was independently obtained by Saffman[10] through an approach based on consideration of the kinetic energy and momentum balance of the fluid. The interaction of two inviscid vortex rings was studied by Sommerfield[11]; the viscous case was examined analytically by Gunzburger[12] and experimentally by Fohl and Turner.[13]

The central purpose of this investigation is to model the pressure field near an axially symmetric jet by a train of randomly spaced viscous vortex rings. Liu et al.[14] presented a preliminary version of the model and substantial experimental results to justify and check it. In this work, the mathematical model is substantially refined and improved and used to compute the near-field pressure. These analytic results are then compared to analogous experimental results reported by Maestrello and Fung.[15] Section II contains the detailed formulation of the model, while in Sec. III, the parameters appearing in the model are chosen based on experimental observations. Section IV presents the results of the application of the model and comparisons with experimental results. Before proceeding, it is important to emphasize that the purpose of the model is to simulate the pressure field in the vicinity of the jet and not the detailed flowfield of the jet itself.

II. Formulation of the Mathematical Model

The pressure field in the vicinity of an axially symmetric jet is to be simulated by the pressure field due to a finite collection of coaxial circular vortex rings immersed in a uniform flow. The motion of a particular vortex ring is influenced by the uniform flow, the other vortex rings, and, in addition there is a self-induced motion. Let $[R_j(t), Z_j(t)]$ be the position of the jth vortex ring at a time t in an axially symmetric cylindrical coordination system (r,z) with z coinciding with the axis of symmetry of the rings. Further, let r_{1j} and r_{2j} be the shortest and longest distance from a point of interest $P(r,z)$ to the center of the vortical core of the jth ring (see Fig. 1). Outside this vortical core, the jth vortex ring induces a potential flow with a stream function given by[8]:

$$\psi_j = -\frac{\Gamma_j}{2\pi}(r_{ij}\text{-}r_{2j})\left[K\left(\frac{r_{2j}-r_{1j}}{r_{2j}+r_{1j}}\right)-E\left(\frac{r_{2j}-r_{1j}}{r_{2j}+r_{1j}}\right)\right]$$

where K and E are the complete elliptic integrals of the first and second kind, respectively, and $\Gamma_j(t)$ is the total circulation of the ring. The corresponding induced velocities in

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the r and z directions are, respectively,

$$W_{1j} = \frac{1}{r}\frac{\partial \psi_j}{\partial z} \qquad \text{and} \qquad W_{3j} = -\frac{1}{r}\frac{\partial \psi_j}{\partial r}$$

Let N be the number of vortex rings present at time t. Then, if none of the vortical cores intersect with each other, the motion of the center of the vortical core of the kth ring is given by:

$$\dot{R}_k(t) = \sum_{j=1, j\neq k}^{N} W_{1j}(t, R_k, Z_k) \tag{1}$$

and

$$\dot{Z}_k(t) = \sum_{j=1, j\neq k}^{N} W_{3j}(t, R_k, Z_k) + U + \frac{\Gamma_k}{4\pi R_k}\left\{\ell n\left(\frac{8R_k}{\delta_k}\right) - 0.558\right\} \tag{2}$$

where $\delta_k(t)$ is the effective size of the core of the kth vortex ring. These formulas are extensions of those for a pair of rings given by Gunzburger.[12] The summations appearing in Eqs. (1) and (2) represent the inviscid flow present at (R_k, Z_k) due to the other $N-1$ vortices. The second term of Eq. (2) represents the velocity of the uniform stream in which the vortex rings are submerged. The last term in Eq. (2) is the self-induced contribution of the kth ring to the velocity of the center of its vortical core. This term was first derived by Tung and Ting[9] by including the effects of viscosity near the center of the vortical core of a vortex ring. These same researchers showed that the effective radius of the viscous core may be defined by:

$$\delta_k = 2(\nu \tau_k)^{1/2} \tag{3}$$

where ν is the kinematic viscosity and τ_k is a time scale defined by:

$$\tau_k(t) = \int_{t_k}^{t} \frac{R_k(s)}{R_k(t)}\,ds \tag{4}$$

where t_k is the time at which the kth vortex ring was created. Tung and Ting[9] also showed that Eqs. (1-3) are valid whenever δ_k is small compared to R_k.

If the point $P(r,z)$ is outside all the viscous cores, the velocity potential due to the vortex rings is given by[8]:

$$\phi = -\frac{1}{2}\sum_{j=1}^{N}\Gamma_j R_j \int_0^{\infty}[\mathrm{sgn}(z - Z_j) \times \exp(-k|z - Z_j|) J_0(kr) J_1(kR_j)]\,dk$$

where J_0 and J_1 are Bessel functions of order 0 and 1, respectively, and where the sgn function is 1 for $z > z_j$ and -1 for $z < z_j$. The corresponding pressure variation is given by the unsteady Bernoulli equation:

$$-\frac{\Delta p}{\rho} = \frac{p_\infty - p}{\rho} = \frac{\partial \phi}{\partial t} + \frac{1}{2}\left[\left(U + \sum_{j=1}^{N} W_{3j}\right)^2 + \left(\sum_{j=1}^{N} W_{1j}\right)^2\right] - \frac{1}{2}U^2 \tag{5}$$

where ρ is the density of the fluid (assumed to be constant) and where it has been assumed that at a distance sufficiently far away from the vortex rings, the flow is unperturbed and has a pressure given by p_∞.

The theoretical model is based on Eqs. (1-5). The instantaneous positions of the vortex rings and the induced pressure fluctuations can be obtained by the numerical integration of these equations. A satisfactory simulation of the pressure field outside (but near) a jet will be accomplished by properly choosing the parameters and functions appearing in the governing equations. Specifically, the uniform velocity U, the creation (or shedding) times t_k, the circulations $\Gamma_k(t)$, and initial conditions for (R_k, Z_k) must be chosen. The parameters are chosen using the experimental observations made in the relevant works mentioned in Sec. I.

III. Choosing the Parameters in the Model

Measured pressure signals are obtained from the experiments described by Maestrello and Fung.[15] The experimental setup (Fig. 2) consisted of a jet nozzle pointing upward and an array of twelve microphones located just outside the boundary of the jet. Typical experimental results which are to be simulated are shown in Fig. 3. In the present simulation, the region of interest is where the experimental measurements were taken. Clearly, this region will be outside the vortical cores of all the vortex rings; thus, the use of the inviscid formula, Eq. (5), for the pressure is justified. The remainder of this section is devoted to the choice of the parameters in the model.

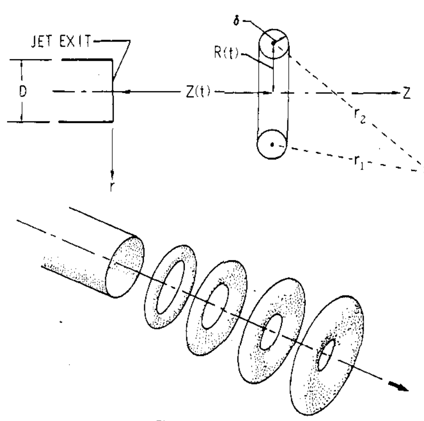

Fig. 1 Geometry of vortex rings.

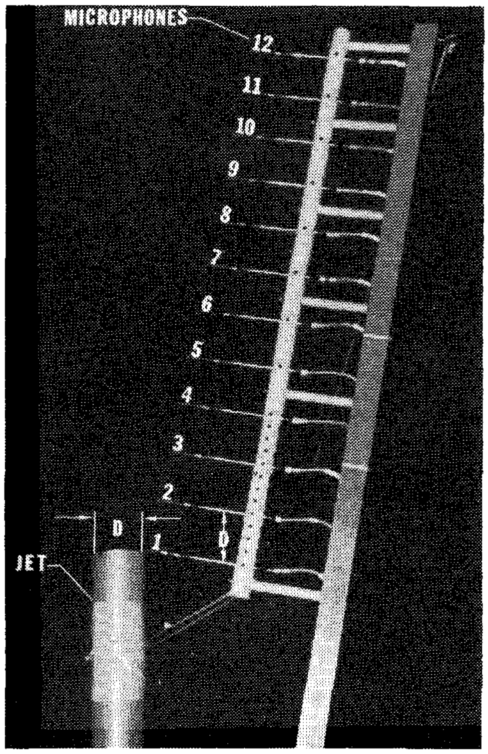

Fig. 2 Experimental setup.

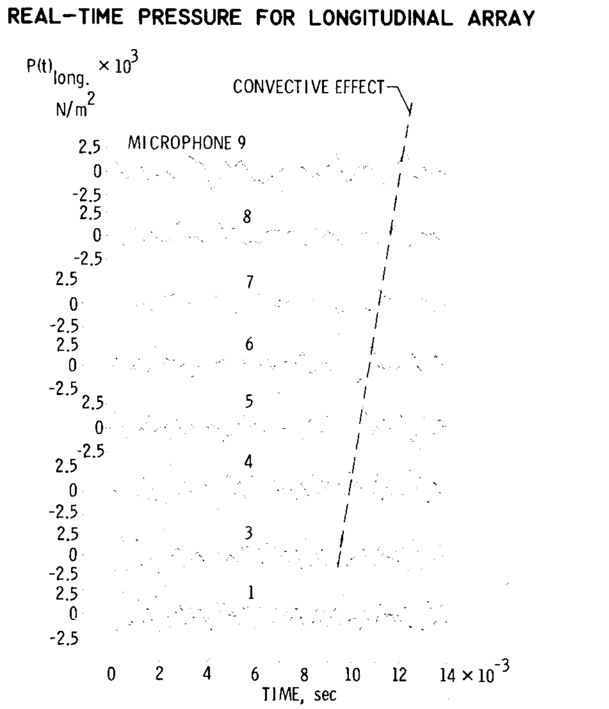

Fig. 3 Time-history of the measured pressure signals at several stations.

Uniform Flow

Let the velocity of flow at the jet exit be denoted by U_e. This is, of course, also the flow velocity in the potential core of the jet. Outside the jet, the flow velocity vanishes. In between, in the shear layer, the average velocity is given by $(U_e/2)$. Since the jet noise is largely generated in the shear layer near the jet exit, it is reasonable to assume that the noise-generating structures will be convected at an average velocity given by the average velocity in the shear layer. Therefore, in our model the velocity of the uniform flow in which the vortex rings are submerged is taken to be $(U_e/2)$.

Random Shedding Times

Crow and Champagne[3] noted that the large-scale vortex-like structures in a jet were generated randomly in time, an observation which is consistent with the random nature of the measured pressure field of Fig. 3. Using data such as those of Fig. 3, a statistical distribution of the time interval between the shedding of successive vortices may be determined. In the present model, the most probable time interval between the shedding of two successive vortices was taken to be the time interval obtained by extrapolating the zero-crossing statistical information on the most probable time intervals of the experimental data at different downstream stations. Based on this most probable time interval, a numerical program was developed which generated random shedding time intervals with a distribution matching that obtained by extrapolation. The test of the validity of the model is based largely on statistical comparisons of the pressure at downstream stations. In other words, by imposing on the model certain statistical data near the jet exit, the model will produce, downstream of the jet exit, statistical data that are in accord with the analogous experimental data.

Circulation

Suppose that the simulation is started at $t=0$ and that $t_k>0$ is the shedding time of the kth vortex. For $t \leq t_k$, the circulation of this vortex is assumed to vanish. Then, at the shedding time $t=t_k$, the kth ring starts rolling up, a process which lasts until the shedding time of the next vortex, i.e., $t=t_{k+l}$. During the rolling up process, the circulation is

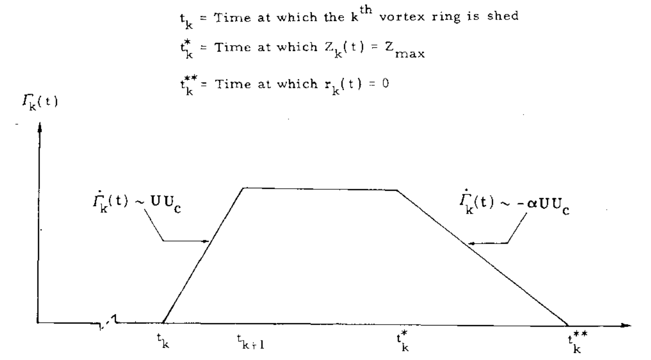

Fig. 4 Time-history of the circulation of the kth vortex ring.

assumed to be determined by the strength of the shear layer U acting over the distance traveled by the developing vortex, $U_c(t-t_k)$. Here U_c is the average convection velocity of the ring with the value equal to 0.6-0.7 of the jet exit velocity and is assumed to be a constant for simplicity. At $t=t_{k+l}$, the vortex becomes saturated and no further increase in the circulation of the kth ring takes place; in fact, the $(k+1)$th vortex then starts to form. During an ensuing period for which $t>t_k$, the kth vortex ring moves downstream with a constant circulation.

Experimental results indicate that vortex ring-type structures and their corresponding pressure signals are seldom observed beyond the potential core. According to Becker and Massaro[2] and Crow and Champagne,[3] vortex-type patterns start dissipating no further than 4 diam downstream from the jet exit. The present simulation emulates this behavior by requiring the circulation to decay linearly in time after a vortex has passed a predetermined station downstream of the jet exit. The position of this station and the decay rate, denoted by z_{max} and α, respectively, are chosen to match the experimental evidence reported in Ref. 15. The values $z_{max}=3D$ and $\alpha=1/10$ have provided the present simulation. Here D is the jet exit diameter. For $Z>Z_{max}$, the circulation will continue decreasing; when the value of the circulation reaches zero, the vortex ring effectively vanishes.

In summary, the circulation of the kth vortex ring is described by the equation:

$$\Gamma_k(t) \sim UU_c f_k(t)$$

where

$$f_k(t)=$$

$$\begin{cases} 0 & 0 \leq t < t_k \\ t-t_k & t_k \leq t < t_{k+l} \\ t_{k+l}-t_k & t_{k+l} \leq t < t_k^*, \text{ where } Z_k(t_k^*)=Z_{max} \\ t_{k+l}-t_k-\alpha(t-t_k^*) & t_k^* \leq t < t_k^{**}=t_k^*+(t_{k+l}-t_k)/\alpha \\ 0 & t_k^{**} \leq t < \infty \end{cases}$$

A sketch of the time history of the circulation is presented in Fig. 4.

It is important to note that since the shedding times t_k are randomly determined, the number of active vortices (those with nonvanishing circulation) varies in time. Furthermore, the strength of vortices produced will also vary. This results in a computational pressure signal which not only has a randomly distributed interval between peaks (i.e., frequency), but also a randomly distributed size of the peaks (i.e., amplitude).

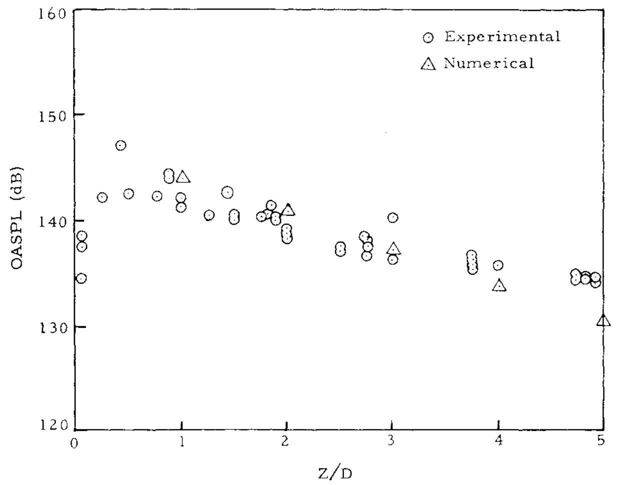

Fig. 5 Overall sound pressure level as a function of downstream positions.

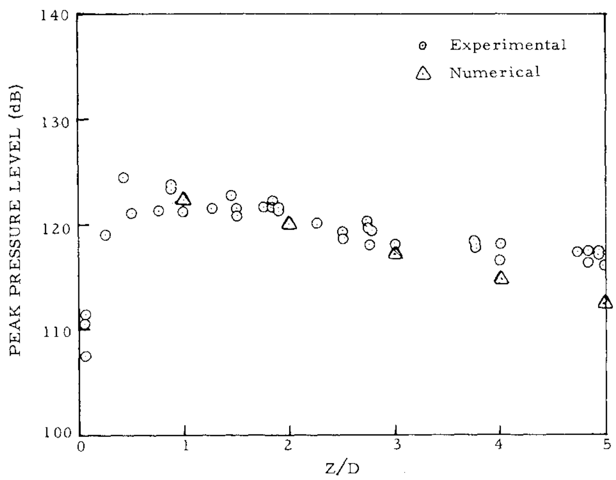

Fig. 6 Peak pressure level as a function of downstream positions.

Initial Conditions

The kth vortex ring is assumed to be created at the jet exit at a time $t=t_k$. Therefore, the initial conditions on (R_k, Z_k) are given by:

$$R_k(t_k)=D/2$$

and

$$Z_k(t_k)=0$$

where D is the jet exit diameter.

IV. Results, Comparisons, and Comments

The experimental conditions were $U_e=240$ m/s and $D=0.062$ m. All numerical and experimental results presented in this section are for stations corresponding to the microphone locations in Fig. 2. Note that these stations are in the near field but outside the shear layer of the jet. No comparison beyond the potential core was made since vortex ring-type structures are rarely seen there.

A preliminary version of qualitative comparisons of the computational and experimental pressure signals was given in Ref. 16. Direct real-time comparisons of pressure signals are impossible because of the random nature of these signals. In fact, such direct comparisons are useless even for two sections of a record obtained from the same experiment. Clearly, only statistical informtion about the pressure signals in both the amplitude and frequency domain are amenable to direct comparisons. Statistical information on the power spectral density functions calculated from the numerical pressure signals at different downstream stations serve as a basic structure of the present comparison. In Figs. 5 and 6 the overall sound pressure level (OASPL) and the peak pressure level are given as a function of downstream positions. Both numerical results show good agreement with those of the experiments at stations up to 3 diam from the jet exit, but deteriorate somewhat at 4 and 5 diam downstream, indicating that the vortex model predicts a too rapid decay of the pressure amplitudes. An account of the Strouhal number based on the peak frequency of the pressure signals is given in Fig. 7 showing that the vortex model provides good prediction in the frequency domain.

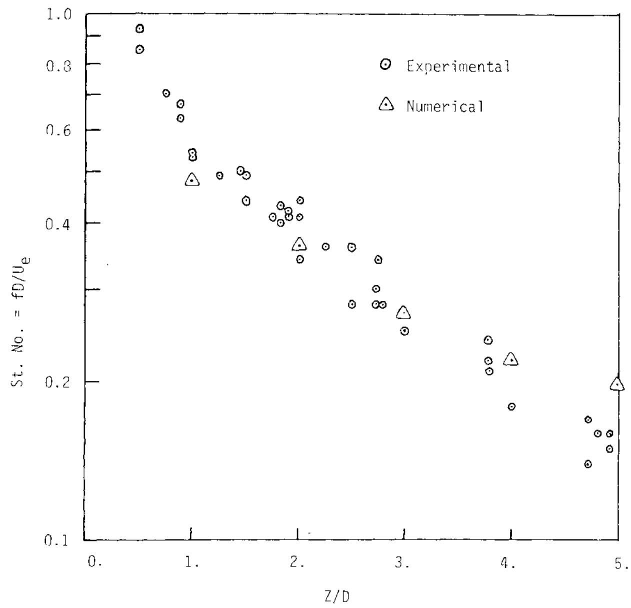

Fig. 7 Strouhal number based on the peak frequency of the pressure signals at several stations.

V. Remarks

The simulation of the pressure field near a jet is accomplished by a train of randomly shed vortex rings. The computational work necessary to compute the pressure field consists of only the solution of two ordinary differential equations, Eqs. (1) and (2), an integral, Eq. (4), and the various quadratures and summations involved with Eq. (5). This is a rather inexpensive procedure on modern computers. Furthermore, randomness is introduced into the model in a very natural way, i.e., by the use of random intervals between the shedding of successive vortices. All free parameters in the model are chosen based on global or statistical observations of jets. Various improvements of the model can be envisioned. For instance, observations indicate that the vortex ring-type structures present in a jet may not be planar. This is especially true downstream of the potential core. The introduction of nonplanar rings could improve the simulation. Of course, these rings would, on the average, be planar, since it is an axially symmetric jet that is being considered. In fact, the deviations of the geometries of the rings from a plane would, in themselves, be random. Other improvements in the simulation may be possible by using more realistic inviscid flowfields (instead of the uniform flow U) and more realistic time histories for the circulation.

It must be noted that no effort was made to satisfy the boundary conditions at the jet exit, i.e., at the nozzle. In fact, the simulation considered in this work ignores interactions with the nozzle by imposing a smooth rolling-up process for the vortices. This results in inaccurate radiated noise predictions in the high end of the frequency spectrum.

Therefore, another improvement in the simulation could be effected by improving the model in the neighborhood of the jet exit.

VI. Acknowledgment

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References

[1]Mollo-Christensen, E., "Jet Noise and Shear Flow Instability Seen From an Experimenter's Viewpoint," *Journal of Applied Mechanics, Transactions of ASME*, Series E, 1967, pp. 1-7.

[2]Becker, H. A. and Massaro, T. A., "Vortex Evolution in a Round Jet," *Journal of Fluid Mechanics*, Vol. 31, 1968, pp. 435-448.

[3]Crow, S. C. and Champagne, F. H., "Orderly Structure in Jet Turbulence," *Journal of Fluid Mechanics*, Vol. 48, 1971, pp. 547-591.

[4]Lau, J. C., Fisher, M. J., and Fuchs, H. V., "The Intrinsic Structure of Turbulent Jets," *Journal of Sound and Vibration*, Vol. 22, 1972, pp. 379-406.

[5]Laufer, J., Kaplan, R. E., and Chu, W. T., "On the Generation of Jet Noise," Noise Mechanism, AGARD-CP-131, 1973, pp. 21-1-21-6.

[6]Hardin, J. C., "Analysis of Noise Produced by an Orderly Structure of Turbulent Jets," NASA TN D-7242, 1973.

[7]Lau, J. C. and Fisher, M. J., "The Vortex-Street Structure of "Turbulent" Jets. Part 1," *Journal of Fluid Mechanics*, Vol. 67, pp. 299-337.

[8]Lamb, H., *Hydrodynamics*, Dover Publications, Inc., New York, 1932.

[9]Tung, C. and Ting, L., "Motion and Decay of a Vortex Ring," *The Physics of Fluids*, Vol. 10, 1967, pp. 901-910.

[10]Saffman, P. G., "The Velocity of Viscous Vortex Rings," *Studies in Applied Mathematics*, Vol. XLIX, 1970, pp. 371-380.

[11]Sommerfeld, A., *Mechanics of Deformable Bodies*, Academic Press, New York, 1950.

[12]Gunzburger, M. D., "Motion of Decaying Vortex Rings with Nonsimilar Vorticity Distributions," *Journal of Engineering Mathematics*, Vol. 6, 1972, pp. 53-61.

[13]Fohl, T. and Turner, J. S., "Colliding Vortex Rings," *The Physics of Fluids*, Vol. 18, 1975, pp. 433-436.

[14]Liu, C. H., Maestrello, L., and Gunzburger, M. D., "Simulation by Vortex Rings of the Unsteady Pressure Field Near a Jet," *Progress in Astronautics and Aeronautics—Aeroacoustics: Jet Noise, Combustion and Core Engine Noise*, edited by I. R. Schwartz, Vol. 43, AIAA, New York, pp. 47-64.

[15]Maestrello, L. and Fung, Y. T., "Quasi-Periodic Structure of a Turbulent Jet," scheduled to appear in *Journal of Sound and Vibration*, May 1979.

[16]Fung, Y. T. and Liu, C. H., "Vortex Simulation of the Pressure Field of a Jet," NASA TM X-73984, 1976.